Optimal Sensor Placement on Highway Networks: A Traffic Dynamics Based Approach

CENTER FOR TRANSPORTATION RESEARCH

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Introduction

- Traffic sensors such as inductive loop detectors are expensive to install and maintain which makes them infeasible for installation throughout the network and covering all segments.
- This poses a problem for traffic management operation and controls that require the knowledge of traffic states on all segments.
- This work focuses on optimal sensor placement to achieve full observability of the system while maintaining a balance between the number of sensors and the degree of observability of the system.
- Consequently, this results in accurate traffic density estimates on segments where the sensors are not installed.

Research Goals

- To present a discrete-time nonlinear state-space model for highway networks having multiple on- and off-ramps based on the Asymmetric Cell Transmission Model.
- To construct a traffic sensor placement problem using the concept of observability for nonlinear systems which is equivalent to maximizing the determinant or trace of observability Gramian matrix given the number of allocated traffic sensors.

Traffic Dynamical System

- The first-order Lighthill-Whitham-Richards (LWR) model is utilized to describe the traffic dynamics. The relationship between the traffic density and flux is given by the triangular fundamental diagram which is defined as:

$$q(\rho(t,d)) = \begin{cases} v_f \rho(t,d), & \text{if } 0 \le \rho(t,d) \le \rho_c \\ w_c(\rho_m - \rho(t,d)), & \text{if } \rho_c \le \rho(t,d) \le \rho_m. \end{cases}$$

where t and d denote the time and space, ρ denotes the traffic density, q denotes the traffic flux, and v_f, w_c, ρ_c and ρ_m are parameters of the fundamental diagram.

- The highway is divided into segments of length l. Given time-step duration T, the discrete-time flow conservation equation for any highway segment with both on- and off-ramp can then be written as

$$\rho_i[k+1] = \rho_i[k] + \frac{T}{l}\left(q_{i-1}[k] + r_i[k] - q_i[k] - s_i[k]\right)$$
$$q_i[k] = \min\left(\delta_i[k], \sigma_{i+1}[k]\right)$$

where δ_i and σ_i denote the demand and supply of Segment i. $r_i[k]$ and $s_i[k]$ are defined similar to $q_i[k]$.

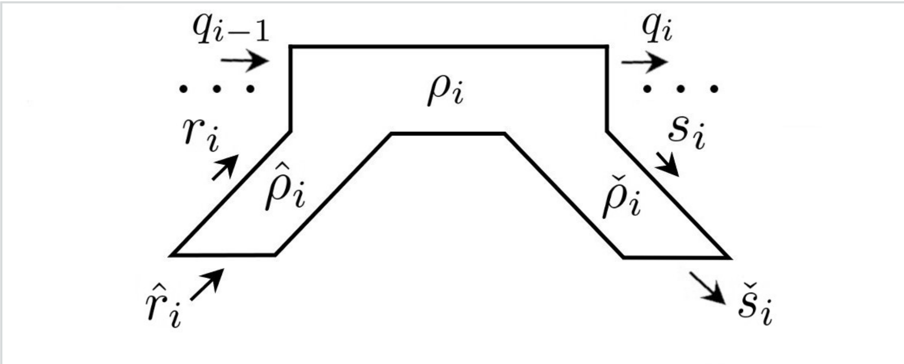

Fig 1. Highway segment connected to on-ramp and off-ramp.

- The state-space and measurement equations for the system respectively are

$$\boldsymbol{x}[k+1] = \boldsymbol{A}\boldsymbol{x}[k] + \boldsymbol{G}\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u},k) + \boldsymbol{B}_u\boldsymbol{u}[k] \quad (1a)$$
$$\boldsymbol{y}[k] = \boldsymbol{\Gamma}\boldsymbol{C}\boldsymbol{x}[k]. \quad (1b)$$

where $\mathbf{x}$ is the state vector containing the traffic densities, $\mathbf{u}$ is the input vector containing demands and supplies of the input and output segments, $\mathbf{A}$ represents the linear dynamics of the system, $\mathbf{B}_u$ represents the effect of the inputs on the system, $\mathbf{f}$ represents the nonlinearities, and $\mathbf{G}$ represents the distribution of the nonlinearities; $\boldsymbol{y}$ is the vector of measured densities, $\boldsymbol{C}$ is the output matrix considering availability of p sensors, and $\boldsymbol{\Gamma} := \mathrm{Diag}(\gamma)$ with $\gamma \in \{0,1\}^p$ represents the selection of sensors–that is, $\gamma_i = 1$ if Segment i is measured and $\gamma_i = 0$ otherwise.

Sensor Placement Methodology

Optimization Problem for Sensor Placement

The traffic sensor placement problem is formulated using the concept of observability based on moving horizon estimation developed in [1]. The following optimization problem is solved to obtain the optimal set of segments to measure out of p possible segments.

$$(\mathbf{P1}) \quad \kappa = \min_{\gamma} \begin{cases} -\det\left(\mathbf{W}_o(\gamma, \hat{\mathbf{x}}_0)\right), \\ -\mathrm{trace}\left(\mathbf{W}_o(\gamma, \hat{\mathbf{x}}_0)\right), \end{cases} \quad (2a)$$
$$\text{s.t.} \quad \gamma \in \mathcal{G}_\gamma, \ \gamma \in \{0,1\}^p. \quad (2b)$$

Here, $\hat{\mathbf{x}}_0$ is the assumed initial state of the system, and $\mathbf{W}_o$ is the N-step observability Gramian which is obtained using the dynamics (1a) and measurement model (1b).

Quality of Sensor Placement Solution

The relative error between the actual and estimated initial states, denoted by ζ, is used to determine the quality of the sensor placement solution γ given by **P1**. The following problem is solved to obtain an estimate of the initial state $\tilde{\boldsymbol{x}}_0$.

$$(\mathbf{P2}) \quad \min_{\tilde{\boldsymbol{x}}_0} \ \|\tilde{\boldsymbol{y}}_\gamma - \boldsymbol{g}_\gamma(\tilde{\boldsymbol{x}}_0)\|_2^2 \quad (3a)$$
$$\text{s.t.} \quad \mathbf{0} \le \tilde{\boldsymbol{x}}_0 \le \mathbf{1} \times \rho_m. \quad (3b)$$

Here, $\tilde{\boldsymbol{y}}_\gamma$ is a stacked N-step measurement vector, and $\boldsymbol{g}_\gamma$ is a function mapping the initial state vector to the stacked output. The relative error is calculated as

$$\zeta := \frac{\|\tilde{\boldsymbol{x}}_0 - \boldsymbol{x}_0\|_2}{\|\boldsymbol{x}_0\|_2}. \quad (4)$$

Case Study

Observability Analysis for Traffic Sensor Placement

- We solve **P1** and **P2** for an arbitrary highway of length 3.1 miles with 20 mainline segments, 2 on-ramps and 2 off-ramps, with different objective functions, observation window lengths, and number of sensors.

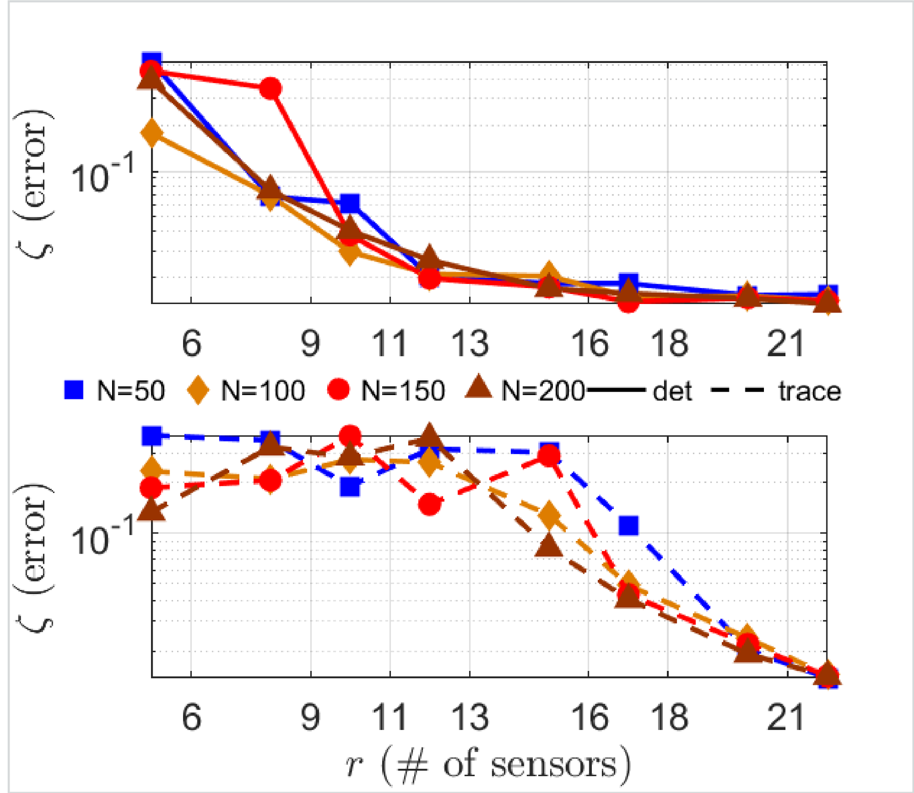

Fig 2. ζ for different number of sensor allocations.

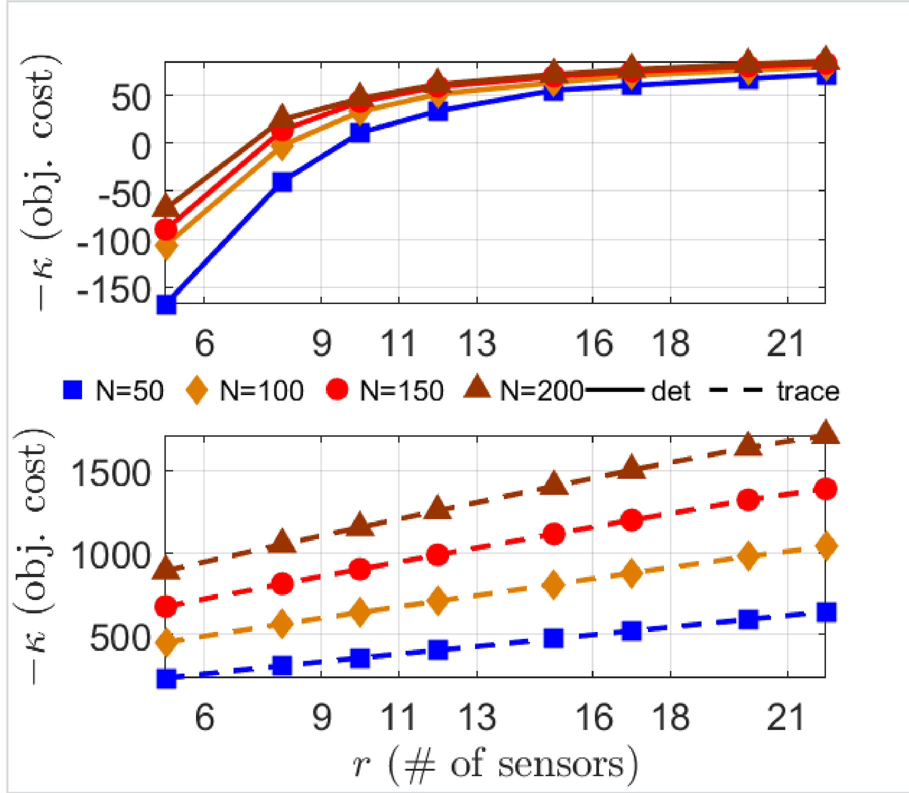

Fig 3. $-\kappa$ for different number of sensor allocations.

- We solve **P1** and **P2** with different assumed initial states $\hat{\boldsymbol{x}}_0$, actual initial states $\boldsymbol{x}_0$, objective functions and number of sensors.

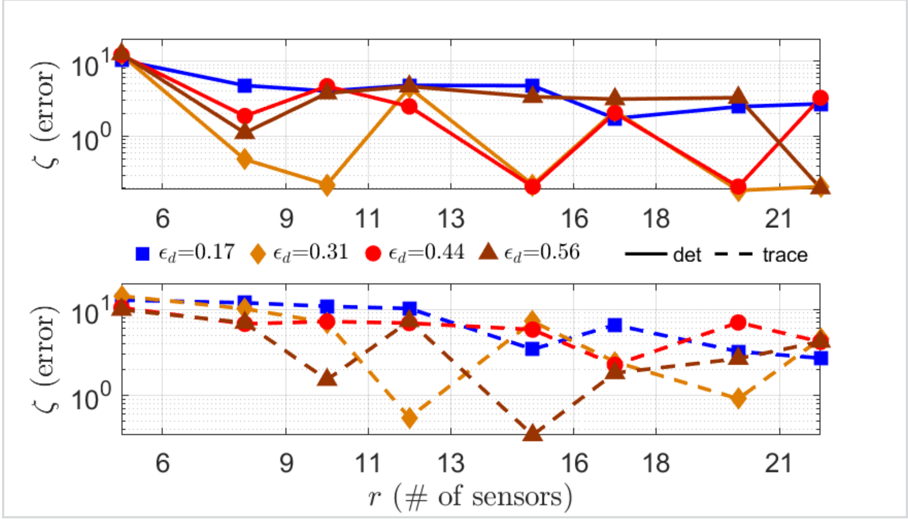

Fig 4. ζ for various presumed initial states.

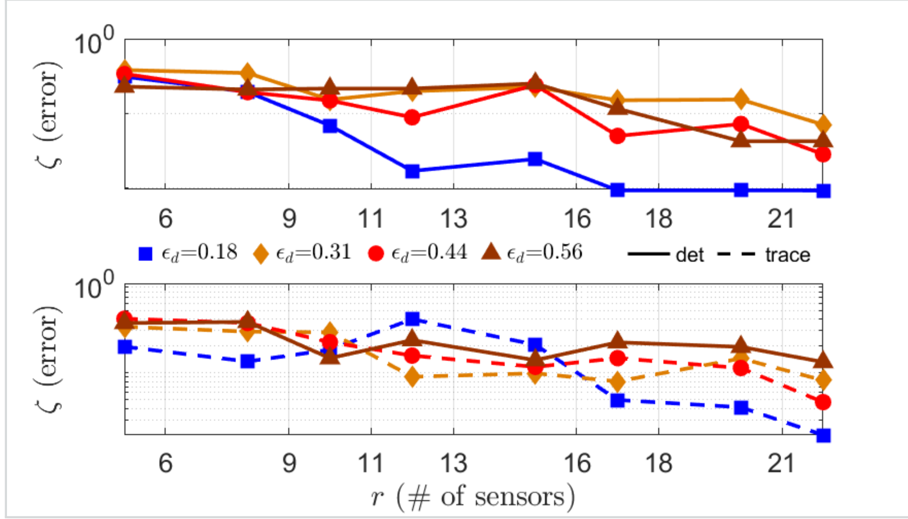

Fig 5. ζ for various actual initial states.

Traffic Density Estimation with Different Sensor Allocations

- We analyse the state estimation error using Extended Kalman Filter (EKF) for different objective functions and number of sensors, and compare the state estimation with randomized and uniform placement of sensors.

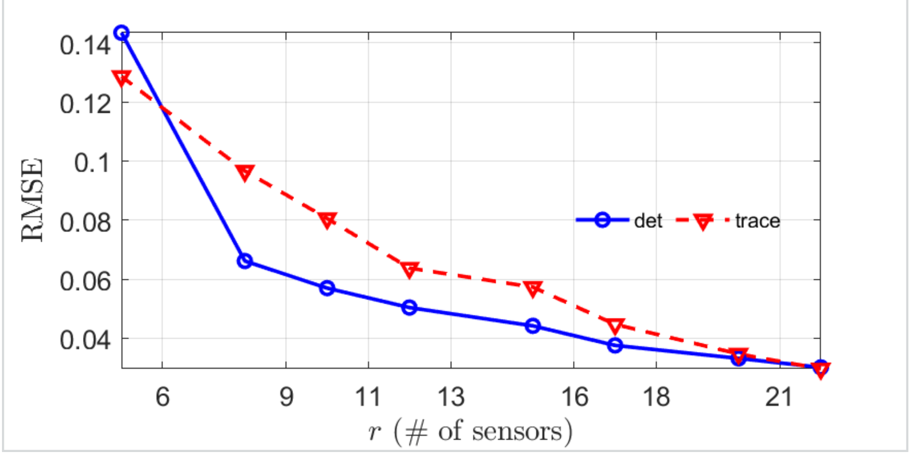

Fig 6. State estimation RMSE with EKF for different sensor allocations.

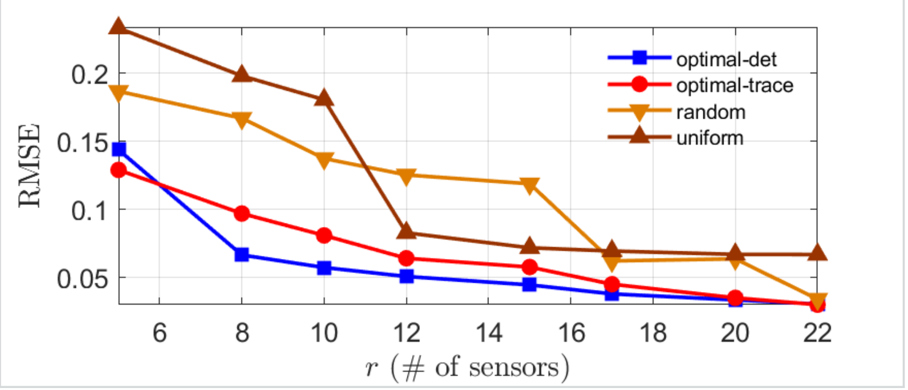

Fig 7. State estimation RMSE with EKF for optimal, randomized and uniform sensor placement.

Key Takeaways

- The proposed optimal sensor placement outperforms randomized and uniform sensor placement in estimating traffic density on highways.
- It allows for the optimal usage of limited resources in terms of sensor allocation for the purpose of traffic density estimation which is important for traffic control.
- The proposed methodology can be used over a wide range of operating conditions.
- The given placement approach can also be utilized with other models in transportation systems and beyond stretched highways, assuming that a nonlinear state-space representation is possible.

Acknowledgement

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References

[1] A. Haber, F. Molnar, and A. E. Motter, “State observation and sensor selection for nonlinear networks,” *IEEE Transactions on Control of Network Systems*, vol. 5, no. 2, pp. 694-708, June 2018.