Activity-Based Travel Demand Modeling for Metropolitan Areas in Texas: Model Components and Mathematical Formulations

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The goal of this project is to comprehensively model the activity-travel patterns of workers as well as non-workers in a household. The activity-travel system will take as input various land use, socio-demographic, activity system, and transportation level-of-service attributes. It will provide as output the complete daily activity-travel patterns for each individual in the household. In addition to the short-term activity-travel decisions, longer-term decisions about household location, employment, and auto ownership will also be considered. The implementation procedure will recognize the dynamic land use-transportation interactions. This report presents an overall conceptual framework for integrated land use-transportation modeling. The modeling of short-term activity-travel decisions and medium-term household decisions is discussed in detail. A conceptual framework is developed and is followed by representation frameworks for the modeling of short-term activity-travel patterns that will be implemented in the project. The mathematical structures of the alternate model types that are proposed for use in the project are also presented.

Activity-Based Analysis, Representation Frameworks, Econometric models

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CHAPTER 1. INTRODUCTION

Since the beginning of civilization, the viability and economic success of communities have been, to a major extent, determined by the efficiency of the transportation infrastructure. To make informed transportation infrastructure planning decisions, planners and engineers have to be able to forecast the response of transportation demand to changes in the attributes of the transportation system and changes in the attributes of the people using the transportation system. Travel demand models are used for this purpose; specifically, travel demand models are used to predict travel characteristics and usage of transport services under alternative socioeconomic scenarios, and for alternative transport service and land use configurations.

The need for realistic representations of behavior in travel demand modeling is well acknowledged in the literature. This need is particularly acute today as emphasis shifts from evaluating long-term investment-based capital improvement strategies to understanding travel behavior responses to shorter-term congestion management policies such as alternate work schedules, telecommuting, and congestion pricing. The result has been an increasing realization in the field that the traditional statistically oriented trip-based modeling approach to travel demand analysis needs to be replaced by a more behaviorally oriented activity-based modeling approach.

1.1 TRIP-BASED APPROACH

The trip-based approach uses individual trips as the unit of analysis and usually includes four sequential steps: trip generation, trip distribution, mode choice, and traffic assignment. The time of day when trips occur is either not modeled or is modeled in only a limited way in the trip-based approach. Most commonly, time is introduced by applying time-of-day factors to 24-hour travel volumes at the end of the traffic assignment step or at the end of the trip generation step. The behavioral inadequacy of the trip-based approach, and the consequent limitations of the approach in evaluating demand management policies (Gordon et al. 1988; Lockwood and Demetsky 1994; Hanson 1980), has led to the emergence of the activity-based approach to demand analysis.
1.2 ACTIVITY-BASED APPROACH

The activity-based approach to travel demand analysis views travel as a demand derived from the need to pursue activities distributed in space (Jones et al. 1990; Axhausen and Gärling 1992). The approach adopts a holistic framework that recognizes the complex interactions in activity and travel behavior. The conceptual appeal of this approach originates from the realization that the need and desire to participate in activities is more basic than the travel that some of these participations may entail. By placing primary emphasis on activity participation and focusing on sequences or patterns of activity behavior (using the whole day or longer periods of time as the unit of analysis), such an approach can address congestion-management issues through an examination of how people modify their activity participations (for example, would individuals substitute more out-of-home activities for in-home activities in the evening if they arrived early from work due to a work-schedule change?).

The shift to an activity-based paradigm has also received an impetus because of the increased information demands placed on travel models by the 1990 Clean Air Act Amendments (CAAAs). These amendments require the inclusion of transportation control measures (TCMs) in transportation improvement programs for MPOs in heavily polluted non-attainment areas and, by state law, for all non-attainment areas in California. Some TCMs, such as high occupancy vehicle (HOV) lanes and transit extensions, can be represented in the existing modeling framework; however, non-capital improvement measures such as ridesharing incentives, congestion pricing, and employer-based demand management schemes cannot be so readily represented (Deakin 1993). The ability to model both individual activity behavior and interpersonal linkages between individuals, a core element of activity modeling, is required for the analysis of such TCM proposals. The CAAAs also require travel demand models to provide (for the purpose of forecasting mobile emission levels) link flows at a high level of resolution along the time dimension (for example, every 30 minutes or an hour as opposed to peak-period and off-peak-period link flows) and to provide the number of new vehicle trips (i.e., cold starts) that begin during each time period. Because of the simplistic “individual-trip” focus of the trip-based models, they are not well equipped to respond to these new requirements (Cambridge Systematics Inc. 1994). Since the activity-based approach adopts a richer, more holistic approach
with detailed representation of the temporal dimension, it is better suited to respond to the new requirements.

Activity-based travel analysis has seen considerable progress in the past couple of decades. Several studies have focused on the participation of individuals in single activity episodes, along with one or more accompanying characteristics of the episode, such as duration, location, or time window of participation. The effect of household interdependencies on individual activity choice is represented in these models in the form of simple measures such as presence of a working spouse, the number of adults, and household structure.

Significant attempts have also been made to broaden the scope of earlier studies to examine activity episode patterns; that is, multiple activity episodes and their sequence over a particular time-span, typically a day. Some of these studies focus only on activity episode scheduling and consider the generation of activity episodes and their attributes as exogenous inputs. Other studies analyze both activity episode generation and scheduling, yielding more comprehensive activity-travel models. Such comprehensive models can potentially replace the conventional trip-based travel demand models (see Guo and Bhat 2001 for a detailed review of the state of the art in activity-based research).

1.3 RESEARCH OBJECTIVES

The current project aims to advance the state of the art in daily activity-travel modeling. It represents one of the first attempts to comprehensively model the activity-travel patterns of workers as well as non-workers in a household. The activity-travel system will take as input various land use, socio-demographic, activity system, and transportation level-of-service attributes. It will provide as output the complete daily activity-travel patterns for each individual in the household. In addition to the short-term activity-travel decisions, longer term decisions about household location, employment and auto ownership will also be considered. The implementation procedure will recognize the dynamic land use-transportation interactions.

Within the broader context of the research objective of the project, this report presents an overall conceptual framework for integrated land use-transportation modeling. The modeling of short-term activity-travel decisions and medium-term household decisions is discussed in detail. A conceptual framework is developed and is followed by representation frameworks for the
modeling of short-term activity-travel patterns that will be implemented in the project. The mathematical structures of the alternate model types that are proposed for use in the project are also presented.

1.4 OUTLINE OF THE REPORT

This report is organized as follows. The next chapter discusses land use-transportation interactions and presents a conceptual framework for an integrated land use-transportation modeling system. Chapter 3 focuses on the modeling of medium-term decisions about household location, auto ownership and employment. Chapter 4 discusses the modeling of short-term activity-travel decisions. Chapter 5 presents the mathematical model structures for the different types of models. Chapter 6 provides the summary and conclusions.
CHAPTER 2. THE PROPOSED INTEGRATED LAND USE - TRANSPORTATION MODELING SYSTEM

This chapter provides an overview of the influence of land use-transportation interactions and the interaction between different decision-making agents on the observed activity-travel patterns of individuals. A conceptual framework is proposed to model the different interactions. The advantages of econometric modeling over other modeling methodologies are discussed. Finally, the application of the model set in forecasting mode is described.

2.1 LAND USE AND TRANSPORTATION

It is conventional wisdom that land use and transport are intimately linked (see, for example, Mitchell and Rapkin 1954; Jones et al. 1983; Jones 1990; Banister 1994; Hanson 1996). While land use represents the spatial pattern of urban development and activities, transportation serves as the mechanism for spatial interaction between geographically dispersed activity sites. Existing theories on land use-transportation interaction are largely based on the concept of accessibility, which refers to the ease of movement between places. The level of accessibility within a given region is determined by the structure and capacity of the transportation system as well as the spatial distribution of opportunities for activity participation (i.e., the land use pattern). A reduction in the cost of movement (in terms of either money or time) leads to increased accessibility to activity opportunities and hence an increase in travel demand. As more interaction occurs, the land use pattern changes because more activities are generated and relocated to places that become more accessible. On the other hand, the location of activities in space affects individuals’ activities and, in turn, travel patterns and the transportation system.

2.2 UNDERLYING DECISION MAKERS

The previous section describes interaction between land use and transportation at an aggregate level. The description, however, does not focus on the underlying decisions of individual agents that manifest themselves in the form of the aggregate-level land use-transportation relationships.

As shown in Figure 2.1, the decision makers who have influence on the urban environment include the household residents, developers, businesses and institutions. The
interactions among these decision makers are the true determinants of the land use-transportation relationship. The developers convert vacant or developed land into urban activity centers, based on their profitability expectations. The institutions impose constraints on development through policies such as zoning and infrastructure availability. The developers and the institutions therefore largely determine the supply of locations for households and businesses. Businesses and firms are providers of services as well as employment centers. Their locations not only determine the flow of commodities, but also influence where people choose to reside and to conduct business transactions or other activities. Households make long-term decisions such as residential location, work location, and auto ownership based on factors such as housing availability and accessibility to activities sites. Conditional on these long-term decisions, households determine their activity agenda and schedules, which in turn determine their travel.

**Figure 2.1 Interactions among the decision makers that result in the observed land use-transportation interactions.**

### 2.3 A CONCEPTUAL FRAMEWORK FOR INTEGRATED LAND USE-TRANSPORTATION MODELING

This section provides a conceptual framework for integrated land use-transportation modeling (see Figure 2.2). The short-term decisions about daily activity participation form the lowest level of models. These decisions are conditional on the individual and household socio-demographics and the activity-travel environment (i.e., the network configuration of roads, the transit system, and the location of opportunities for activity participation). The socio-demographics and the activity-travel environment are themselves a result of medium-term household decisions, evolution patterns, and policy decisions of institutions.
The medium-term household decisions (such as those about employment, household location, and auto ownership) are modeled at a higher level prior to the modeling of activity patterns. Analogous to the households, firms can also be considered to make medium-term decisions (such as those about establishment location and jobs per establishment). The modeling of firm level decisions is not a primary focus of this work. However, these decisions have to be accommodated in any comprehensive travel demand model system, as the medium-term decisions of household and firms are interdependent (through the job and land market mechanisms).

The medium-term decisions are influenced by socio-demographic and economic evolution processes that take place continuously over time. The demographic transition results from “a complex set of social and demographic changes that include aging, household formation, divorce and household dissolution, mortality, birth of children, migration to and from the region” (University of Washington, 2000). The economic transition captures the growth or decline in the different job sectors. These evolution patterns also influence the land use patterns and the
transportation system characteristics. They are therefore modeled at the highest level prior to the modeling of medium-term decisions.

The activity-travel patterns of individuals lead to the spatial and temporal distributions of travel patterns. These patterns, in turn, influence the medium-term decisions of individuals and firms and the land use and transportation policy decisions.

2.4 PROPOSED APPROACH TO INTEGRATED LAND USE-TRANSPORTATION MODELING

The primary methods that have been employed for the modeling of activity-travel behavior are econometric modeling methods and computational process models (CPMs).

The family of econometric models including discrete choice models, hazard duration models, and limited-dependent variable models remains a powerful approach to activity-travel analysis and has led to several operational model systems. Its strength lies in allowing the examination of alternative hypotheses about the causal relationships among behavioral indicators. A limitation of the econometric approach is that it may not comprehensively represent the decision mechanisms underlying the observed activity and travel choices (Kitamura 1996).

Computational process models have been proposed as an alternative approach to modeling the complex activity-travel behavior. A CPM is basically a computer program implementation of a production system model, which is a set of rules in the form of condition-action (IF-THEN) pairs that specify how a task is solved (Gärling et al. 1994). The modeling approach focuses on the process of decision making and captures heuristics and bounded rationality, as opposed to assuming overriding paradigms such as utility maximization. Hence, the modeling approach potentially offers more flexibility than econometric models in representing the complexity of travel decision making. A major drawback of CPMs, however, is that they lack a statistical error theory, which makes it more difficult to generalize their outcomes and apply them to policy evaluation (Ettema et al. 1996). In addition, the models have very challenging data requirements for model estimation, application, and validation, and the assumptions they make about the search process have not been validated (Bowman and Ben Akiva 1996).
This research effort will use the econometric modeling approach for the modeling of short-term activity-travel patterns and medium-term household decisions. The activity-travel models explore how activity and travel patterns are related to land use and socio-demographic characteristics of the traveler. The medium-term decision models capture the effects of socio-demographics, land use, transportation system, and the activity patterns of individuals on the choices about household location, auto ownership, and employment. The econometric models predict the probability of decision outcomes. In theory, the econometric modeling approach can also be adopted for the modeling of socio-demographic and economic evolution patterns. However, these evolution patterns are too complex to characterize and thus are not modeled at the same level of detail as the medium- and short-term decisions.

2.5 APPLYING THE INTEGRATED LAND USE-TRANSPORTATION MODELING SYSTEM

The application of the econometric model system is based on an “incremental” prediction process. In this approach, the socio-demographics and medium-term decisions, as well as activity-travel patterns, are predicted for a certain time increment beyond the base year (the base year refers to the year for which the socio-demographic and activity-travel characteristics have been synthesized for the entire planning region). Once the demographics, medium-term decisions (such as employment and household location), and the activity-travel pattern of the individuals are predicted for the time increment beyond the base year, those predictions become the basis for the predictions for the next time increment. This incremental prediction process continues until the predictions for the desired target year are obtained. The reader will note that this incremental procedure recognizes the dynamic nature of decision making.

The structure of the incremental forecasting procedure is presented in detail in Figure 2.3. The base year socio-demographics and the activity-travel environment serve as the inputs to the estimated modeling system. The outputs from the activity-travel model represent the activity-travel patterns for the base year. The activity-travel patterns, along with the base year socio-demographics and activity-travel environment, are used to compute the accessibility measures for the base year. Subsequently, the base year socio-demographics, activity-travel environment, and the accessibility measures are used as inputs to the socio-demographic evolution and medium-term decision models to obtain predictions of the socio-demographic and activity-travel
environment characteristics for one time increment beyond the base year. These predictions serve as inputs to the activity-travel model to predict the activity-travel patterns for one time increment beyond the base year. The predicted attributes for one time increment beyond the base year are used to recompute new accessibility measures and predict the socio-demographics and activity-travel environment for two time increments beyond the base year. This process continues until the activity-travel predictions for the target year are obtained.

The application of the activity-travel, socio-demographic evolution, and medium-term decision models in Figure 2.3 has two steps, though this is not explicitly indicated in the figure. In the first step, the estimated models are used to obtain probabilistic predictions for the next time increment. In the second step, these probabilistic predictions are translated into deterministic predictions using a micro-simulation procedure (see Bhat and Misra 2000 for a discussion of such a procedure).
The rest of the report will focus on detailed representation structures for the medium-term decisions and the activity-travel models. The next chapter focuses on medium-term household decisions. Chapter 4 provides the framework for modeling of short-term activity-travel decisions and Chapter 5 provides the mathematical model structures.
CHAPTER 3. MODELING MEDIUM-TERM HOUSEHOLD DECISIONS

There are three major household medium-term decisions that influence land use and travel patterns: housing, work, and auto ownership. The modeling of these decisions is discussed in this chapter.

3.1 HOUSING CHOICE

Housing decisions are often considered as a bundle of related choices, including the decision to move (mobility choice), the selection of tenure, and the selection of dwelling location. From a behavioral point of view, these decisions could be made either sequentially or jointly.

Factors expected to influence the mobility choice include the household income, stage of life cycle, number of full-time workers, and number of years each worker has held their current jobs (Waddell 1996). Older or low-income households are less likely to move. The presence of children might stimulate or inhibit a move, depending on the residential location. The presence of multiple workers in a household may affect mobility in either way. It could be argued that, because of the ripple effects of relocation on all workers, a multi-worker household will have a lower propensity to relocate. Alternatively, one could also argue that more workers in a household means a higher likelihood that one of them will change jobs, resulting in a higher probability of a relocation. The length of employment represents the other link between work and residence. The longer the employment, the less likely it is that a household will move.

Economic theories of housing tenure choice stress the role of the relative costs of rental- and owner-occupied housing, particularly the effect of the tax system on relative costs (Rosen 1979; King 1980). Housing demand is also closely related to tenure choice. People who want better quality housing are more likely to own. Thus, wealthier households are more likely to afford to be able to buy housing and become homeowners. Family background also has an important influence. People whose parents are homeowners are more likely to become homeowners themselves, reflecting either the transfer of resources from parents to their adult children or the influence of the parents’ attitudes toward home ownership (Salvo and Ermisch 1997).
Residential location choice is much more complicated than a simple trade-off between housing and accessibility. Location preferences vary among households with different and even with similar socioeconomic and demographic characteristics. Location choice depends on non-spatial factors such as income of the household and housing and transportation costs. It is also influenced by spatial factors such as the quality of nearby schools and proximity of parks. However, the physical distance has become less and less important with the dispersion of employment centers and increased personal mobility. The information revolution with its computer networking and Internet is fast reducing the dominance of physical distance on housing location selection (Harvey 1991; Dear and Flusty 1998; Phe and Wakely 2000). Other hard-to-quantify factors that also affect location choice include ethnic preferences, racial biases, family loyalty to specific neighborhoods, and preferences for architectural styles. Social status also has a significant role in the households’ decision-making process (Maclennan 1982), especially in societies with a strong stratified structure.

3.2 WORK LOCATION CHOICE

The participation of household members in the labor market is important for the land use-transportation interactions for a couple of reasons. First, individuals supply their time and skills in the labor market in exchange for wages, which form the major source of income used to pay for housing and other goods and services. Second, work represents the most frequent destination of travel other than home.

The monocentric model (Alonso 1964) underlying the standard urban economic theory and the gravity model derivatives originated from Lowry (1964) have both assumed that households’ decision about workplaces and residential locations are not related. This assumption has come under increasing scrutiny (Waddell 1993). Although individuals would not be expected to make simultaneous decisions regarding their residence and work locations, some individuals will make workplace decisions based on predetermined residence locations while others will make residence decisions on the basis of predetermined workplace locations. The degree to which residence location is driven by workplace location, or vice-versa, may vary with the degree to which workplace locations are dispersed in a multinodal city, as well as household tenure, ethnicity, and socioeconomic status (Waddell 1993).
3.3 AUTO OWNERSHIP CHOICE

Vehicle ownership has been treated as an independent choice modeled within travel demand systems and influenced principally by socio-demographic characteristics of households (Waddell 2001). This approach is deficient in accounting for effects of neighborhood characteristics and changes in transit services. Thus, vehicle ownership should be examined together with other medium-term household choices regarding residence and workplace locations.

3.4 MODELING FRAMEWORK

The framework proposed for modeling the above listed medium-term household decisions is based on the random utility theory. In short, households are assumed to make a rational decision by choosing the alternative that has the maximum utility value. For instance, the utility of a residential location is measured by considering the relevant locational attributes as well as household preferences. The neighborhood with the highest utility value will be chosen.

As mentioned earlier, the medium-term household decisions are all inter related. A change in an individual’s workplace location may trigger residential relocation. The choice of tenure has an effect on the type and the location of housing. Decisions about vehicle ownership may be made jointly with residential location choice. In this research, alternate model structures with multiple tiers will be examined, both statistically and behaviorally, to determine the most suitable model for the joint estimation of medium-term household decisions. The outcomes of the model can then be used as exogenous inputs to the modeling of household activity- and travel-decisions.
CHAPTER 4. MODELING SHORT-TERM ACTIVITY-TRAVEL PATTERNS OF INDIVIDUALS

This chapter focuses on the modeling of activity-travel behavior of individuals. The individual and household socio-demographics and the activity-travel environment characteristics are assumed to be exogenous inputs. Medium-term decisions of employment choice, household location choice, and auto ownership are modeled prior to the modeling of activity patterns and consequently are assumed to be inputs to the activity modeling.

This chapter first characterizes a typical activity string of workers and non-workers. This is followed by the development of a conceptual framework that captures the choice hierarchy and the household interactions in the generation, allocation, and scheduling of activities. Finally, representation frameworks for activity-travel patterns are presented, that can be implemented using readily available data from activity-travel surveys.

4.1 WHAT CHARACTERIZES THE DAILY ACTIVITY STRING OF AN INDIVIDUAL?

Individuals make choices about the different activities to be pursued in a day and string them together in an activity-travel pattern. Travel is derived from the need to participate in the desired activities. The objective of this section is to completely characterize the daily activity-travel strings of individuals.

The activity pattern of workers rests on the regularity and the fixity of the work activity. No such obvious fixity is present in the case of non-workers (retired people and homemakers). Recognizing this critical difference, representations are developed separately for workers and non-workers. The activity-travel patterns of students are characterized by the regularity of the school activity, analogous to the fixity of the work activity of the workers. The activity-travel patterns of students can, therefore, be represented by a framework similar to that of workers. Hence, a separate representation for the activity-travel patterns of students is not presented in this report. For both the worker and non-worker representations, we consider 3 a.m. as the beginning of the day and assume that the individual is at home during this time. The following discussion of activity-travel representations for workers and non-workers is drawn from earlier works by Bhat and Singh (2000) and Bhat and Misra (2000).
4.1.1 Activity String of Workers

The daily pattern of workers is characterized by four different sub patterns: a) The before commute pattern, which represents the activity-travel undertaken before leaving home to work; b) The commute pattern, which represents the activity-travel pursued during the home-to-work and work-to-home commutes; c) The work-based pattern, which includes all activity and travel undertaken from work; and d) The post home arrival pattern, which comprises the activity and travel behavior of individuals after arriving home at the end of the work-to-home commute. The home-to-work and work-to-home commutes are grouped into a single commute pattern since the travel mode for both these commutes will, in general, be the same. Within each of the before-commute, work-based, and post home arrival patterns, there might be several tours. A tour is a circuit that begins and ends at home for the before-commute and post home-arrival patterns, and is a circuit that begins and ends at work for the work-based pattern. Further, each tour within the before-commute, work-based, and post home arrival patterns may comprise several activity stops. Similarly, the home-to-work and work-to-home components of the work commute pattern may also comprise several activity stops. Figure 4.1 provides a diagrammatic representation of the worker activity-travel pattern.

The characterization of the complete workday activity-travel pattern is accomplished by identifying a number of different attributes within the representation discussed above. These attributes may be classified based on the level of representation they are associated with: that is, whether they are associated with a pattern, a tour, or an episode. Pattern-level attributes include the number of tours for the before-commute, work-based and post home arrival patterns, and the home-stay duration before the home-to-work commute for the commute pattern. Tour-level attributes include the travel mode, number of stops, and home-stay duration before each tour in the before-commute and post home arrival patterns, work-stay duration before each tour in the work-based pattern, and the sequence of tours in the pattern. Episode-level attributes include activity type, travel time from previous episode, location of episode, activity duration, and the sequence of episode in the tour.
4.1.2 Activity String of Non-Workers

In the case of non-workers, the activity-travel pattern is considered as a set of out-of-home activity episodes (or “stops”) of different types interspersed with in-home activity stays. The chain of stops between two in-home activity episodes is referred to as a tour. The pattern is represented diagrammatically in Figure 4.2. A non-worker’s daily activity-travel pattern is characterized again by attributes associated with the entire daily pattern, a tour in the day, and an episode. Pattern-level attributes include whether or not the individual makes any stops during the day, the number of stops of each activity type if the individual leaves home during the day, and the sequencing of all episodes (both stops and in-home episodes). The only tour-level attribute is the travel mode for the tour. Episode-level attributes include the episode duration, travel time to episode from previous episode (except for the first home-stay episode), and the location of out-of-home episodes (i.e., stops).
4.2 GENERATION-ALLOCATION-SCHEDULING OF DAILY ACTIVITY-TRAVEL DECISIONS WITHIN A HOUSEHOLD: A CONCEPTUAL FRAMEWORK

The decision of individuals to participate in activities is motivated by both individual and household needs. The scheduling of activities of workers is primarily constrained by the need to be at work for a pre-determined period of the day. In addition, the sequencing and scheduling decisions of individuals (both workers and non-workers) are constrained by joint activity participation and the need to share the household autos for trip making. This section develops a framework for the daily generation, allocation, and scheduling decisions that occur at the household level. Figure 4.3 presents this framework diagrammatically.
On any workday, individuals choose to undertake a set of “mandatory” activities. These are typically work-related activities that have to be pursued daily, based on long-term decisions about employment. Conditional on these mandatory activities, a set of personal and household-level “flexible” activities for the day is generated. These include activities such as personal business, shopping, and recreation. Constrained by mandatory and flexible activities that the individuals have decided to undertake, the household activities are distributed among the members of the household. Some activities may require joint participation of the household members. If the household does not have an auto for each adult, decisions about auto use are also made depending on the sets of activities to be pursued by the different individuals during the course of the day. Certain members of the household may choose to undertake additional “serve-passenger” trips to facilitate activity participation of other household members.

The individuals in the household sequence the activities into their activity-travel string while ensuring that the constraints placed by mandatory activities and joint activities are
satisfied. The outcome of this complete generation-allocation-scheduling process is the complete activity-travel pattern of all the individuals in the household. The process essentially defines all the attributes that characterize the activity-travel pattern as it is defined in the previous section. The daily decisions subsequently influence the medium- and long-term decisions, leading to an evolution in the activity participation behavior over time.

4.3 REPRESENTATION FRAMEWORKS FOR MODELING ACTIVITY-TRAVEL PATTERNS OF INDIVIDUALS

The conceptual framework presented above provides a “natural” way to visualize the activity-travel generation of individuals within a household context. The generation-allocation-scheduling approach captures inter personal dependencies in terms of joint activity participation and the delegation of tasks among the members of a household. It also explicitly considers the sharing of autos in making trips.

The implementation of the above approach would require explicit modeling of generation and allocation decisions of the household. Most of the household activities like shopping and recreation are of the “flexible” type and are not pursued on a daily basis. A household’s decision to participate in such an activity on any day depends on the time that has passed since the last participation in the same activity. Hence, modeling these decisions requires data about the activity participation of the household members over several consecutive days. For the current research, only a single-day activity-survey data is available. Therefore, it is not possible to explicitly model the generation-allocation mechanisms. Nonetheless, the interpersonal dependencies within the household need to be considered in the modeling process.

This section provides representation frameworks for modeling the individual activity-travel patterns within the context of a single-day’s survey data. In this proposed approach, the individuals’ decisions are modeled. An individual’s decisions include decisions about all the attributes that characterize the individual’s activity string (e.g., number of tours and number of stops in each tour). In making these choices, the individual is assumed to consider all his or her personal and household needs for the day and recognize the constraints imposed by joint participation needs and auto-sharing requirements. Thus, in the representation frameworks subsequently presented, the generation-allocation mechanism is implicitly incorporated into
individual decisions. For example, if a particular member of a household decides to make a shopping stop, it can be inferred that the household decided to undertake a shopping activity for the day and assigned it to this person.

The nature of interpersonal interactions and hence the complexity of the framework depends, to a large extent, on the composition of the households. It is very difficult to develop a generic representation framework to capture the interactions for all possible household types. Hence, the most common types of household are identified based on an empirical analysis of the households in the Dallas Fort Worth data set (Table 4.1).

<table>
<thead>
<tr>
<th>Category Name</th>
<th>% in the DFW data set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1</td>
<td>20.50%</td>
<td>HH with a single adult, the adult is a worker</td>
</tr>
<tr>
<td>Type 2</td>
<td>10.30%</td>
<td>HH with a single adult, the adult is a non-worker</td>
</tr>
<tr>
<td>Type 3</td>
<td>8.60%</td>
<td>HH with two adults, both are non-workers</td>
</tr>
<tr>
<td>Type 4</td>
<td>22.70%</td>
<td>HH with two adults, one is a worker and the other is not</td>
</tr>
<tr>
<td>Type 5</td>
<td>24.30%</td>
<td>HH with two adults, both are workers</td>
</tr>
</tbody>
</table>

The five types together constitute about 87% of all household types. The following sections describe representation frameworks for modeling the activity patterns of individuals in each of these household types.

4.3.1 A Representation Framework for Individuals in Type 1 Households

The framework developed by Bhat and Singh (2000) can be adapted easily to model this type of household. The representation framework is based on modeling the pattern/tour-level attributes first (Figure 4.4), and then modeling the stop-level attributes (Figure 4.5) conditional on the pattern/tour-level attributes. The number of tours in the before-commute, work-based, and post home-arrival patterns, and the sequence of tours in these patterns, are implicitly modeled in Figure 4.4 by determining if an individual makes a first tour, and then, conditional on making the first tour, if the individual makes a second tour (in concept, the procedure can be extended to more than two tours in a pattern). Similarly, the sequence of stops in a tour is modeled implicitly in Figure 4.5 by determining the characteristics of the first stop, then the second conditional on the first, the third conditional on the first two, and so on.
The commute mode choice and the number of commute stops can be modeled using a joint unordered-ordered discrete choice model system. For example, Bhat and Singh (1997) have recently developed a joint model of mode choice in the evening commute, the number of evening commute stops, and the number of stops in the post-home arrival tour using such a methodology. The duration of home stay before each tour may be modeled using hazard-based
duration models (see Hamed and Mannering 1993 and Bhat 1996 for use of such models to examine activity duration). The joint activity-type choice, activity duration, and travel time duration may be modeled (separately for each of the periods) using a joint discrete/continuous econometric system (see Bhat 1998a for the estimation and application of such a joint model for the evening commute period). The joint modeling approach allows for spatial-temporal interactions in stop-making decisions. The location choice of the stop can be modeled subsequently using disaggregate spatial destination choice models (Fotheringham 1988) by identifying all possible destinations that can be reached by the travel mode assigned for the tour (of which the stop is a part) and within the travel time duration estimated earlier.

**4.3.2 A Representation Framework for Individuals in Type 2 and Type 3 Households**

These types of households are characterized by the absence of workers and children. For the single-adult households with the adult not working (Type 2), the representation framework developed by Bhat and Misra (2000) can be directly applied. The households with two adults; both non-workers (Type 3), may typically represent elderly people living together. The two can be assumed to pursue almost all activities together, such that a single activity string can represent their activity pattern. Hence the Bhat and Misra framework can again be directly applied.

The modeling of a non-worker’s daily pattern is achieved by modeling the pattern-level attributes first, followed by the tour-level attribute of mode choice, and finally the episode-level attributes. This hierarchical approach is adopted because the decisions regarding pattern-level attributes are driven by the basic activity needs of the individual (and the household of which the individual is a part). Consequently, and consistent with the derived demand philosophy of the activity-based approach, the pattern-level decisions are considered to be the highest level of the analysis hierarchy. On the other hand, decisions regarding the episode-level attributes tend to be driven primarily by scheduling convenience, short-term temporal constraints, and travel conditions. Consequently, these attributes are relegated to the lowest level of the analysis hierarchy. The tour-level attribute of travel mode choice is positioned at the intermediate level of the analysis hierarchy since it affects the attributes of all out-of-home episodes within the tour.
The pattern-level attributes are modeled using a system of three model components (Figure 4.6). The first model component, which takes the form of a bivariate binary-ordered response probit formulation, jointly models the decision to make at least one stop (instead of staying at home for the entire day) and the decision of the number of stops if the individual leaves home during the day. The second model component, which uses a multinomial logit formulation for stop type, partitions the total number of stops (determined in the first model component) into the number of stops by each out-of-home activity type. The final model component, which has a multinomial logit form with a pattern string as the unit of analysis, models the number of in-home episodes in an individual’s activity-travel pattern along with the entire sequence of all episodes (in-home and out-of-home) in the individual’s activity pattern, given the number of stops by type in the pattern.

![Figure 4.6 Framework for Modeling the Pattern-Level Attributes](image)

Figure 4.6 Framework for Modeling the Pattern-Level Attributes

Figure 4.7 presents an overview of the four remaining model components used to analyze the tour- and episode-level attributes. The tour travel mode is modeled using a discrete choice formulation. Since the duration of the first home-stay episode is likely to be different from other subsequent home-stay episodes because of life style and sleeping habits, the first home-stay duration is modeled prior to all other episode-level attributes using a hazard model. Next, the travel time to the episode from the previous episode and the activity duration of the episode for all episodes other than the first home-stay episode are modeled jointly. Finally, the spatial location of each out-of-home episode (stop) is modeled using a disaggregate spatial destination choice model.
4.3.3 A Representation Framework for Individuals in Type 4 Households

Type 4 households consist of two adults, one of whom is a worker and the other of whom is not. The framework for the modeling of the activity-travel patterns of individuals in this type of household is drawn primarily from the approaches developed independently for the modeling of the activity-travel patterns of workers and non-workers (Figure 4.8).

The commute mode choice and the number of non-work commute stops made by the worker are modeled first. The commute stops are further classified into personal and serve-passenger stops. The decision of the non-worker to stay at home all day as opposed to making activity stops is then modeled. Conditional on these high-level choices, the worker makes tour-level choices. It is assumed that the worker can engage in activity participation jointly with the non-worker in the household only during the pre-work and post home-arrival periods (hence, the individual pursues none of the work-based activities jointly with other household members). It is also assumed that entire tours are pursued alone or jointly. This adds another tour-level attribute for the before-commute and post home-arrival patterns of workers. If the worker chooses to undertake joint tours, the corresponding times are “blocked out” for the non-worker. The number of “personal” stops and the sequence of all stops are then modeled for the non-worker in the household. The stop-level attributes of the stops the non-worker makes jointly with the worker are inferred from the choices made by the worker. The tour- and stop-level attributes of the non-worker are finally modeled to complete characterizing the activity string of both the worker and the non-worker. In essence, decisions about all the attributes of activities jointly
undertaken are assumed to be made by the worker. The household auto is assumed to be available for the use of the non-worker if the worker does not use it at the given time. School going children, if present in the house, are assumed not to make any travel decisions by themselves. Their choice of mode to school is incorporated into the decision of either the worker or the non-worker.

Figure 4.8 Framework for Modeling the Activity-Pattern of Individuals in Type 4 Households

4.3.4 A Representation Framework for Individuals in Type 5 Households

This type of household is characterized by the presence of two working adults. In the modeling approach, the more constrained of the two workers is first identified. The commute mode choice and the number of commute stops made by this person are first modeled. The commute characteristics of the other worker are modeled next. It is assumed here that the two working adults have their own autos and hence can independently choose the mode for their commute and other trips. The trip-level characteristics of the constrained worker are modeled,
conditional on the commute characteristics of the two workers. Similar to assumptions made in the modeling of activity-patterns of individuals in Type 4 households, joint participation is assumed to be possible only before and after work. Also, entire tours are assumed to be made jointly or independently. The trip-level characteristics of the less-constrained worker are modeled next. Conditional on the trip-level characteristics of the two workers, the stop-level attributes of the stops made jointly are modeled. Subsequently, the stop-level attributes of the personal stops made by the two workers are modeled independently. Again the school going children are not assumed to make any trips by themselves. Their mode choice is incorporated into the decision of either worker.

![Figure 4.9 Representation Framework for Modeling Activity-Travel Patterns of Individuals in Type 5 Households](image)

**Figure 4.9 Representation Framework for Modeling Activity-Travel Patterns of Individuals in Type 5 Households**
CHAPTER 5. ECONOMETRIC MODELS

This chapter provides the mathematical formulations for the different model components that comprise the integrated land use-transportation modeling system described in the previous chapters.

5.1 JOINT UNORDERED-ORDERED DISCRETE CHOICE MODELS

Joint unordered-ordered discrete choice models involve the joint modeling of an unordered choice (e.g., mode choice for a tour or commute) and one or two ordered choices (e.g., number of stops). Bhat and Singh (1997) have adopted this joint unordered-ordered discrete choice economic structure for the joint modeling of work-to-home commute mode choice, the number of non-work commute stops, and the number of stops in the first tour of the post home arrival pattern. The ordered response formulation for the number of stops recognizes the ordinal and discrete nature of stops. The economic structure is presented here in this context.

In the following model structure, index $i$ is used to represent mode ($i=1,2,3…I$), index $k$ to represent the number of non-work work-to-home commute stops ($k=1,2,3…K$), index $l$ to represent the number of post-home arrival stops ($l=1,2,3…L$), and index $q$ to represent the $q$th individual ($q=1,2,3…Q$). The equation system is then as follows:

$$
\begin{align*}
    u_{qi}^* &= \beta_i z_{qi} + \epsilon_{qi}, \text{ mode } i \text{ is chosen if } u_{qi}^* > \max_{j=1,2,I,i} u_{qj}^* \\
    s_{qi}^* &= \gamma_i x_{qi} + \eta_{qi}, s_{qi} = k \text{ if } \delta_{i,k,1} < s_{qi}^* \leq \delta_{i,k} \\
    w_{qi}^* &= \alpha_i y_{qi} + \lambda_{qi}, w_{qi} = l \text{ if } \theta_{i,l,1} < w_{qi}^* \leq \theta_{i,l} \\
\end{align*}
$$

(Eq.1)

$u_{qi}^*$ is the indirect (latent) utility that the $q$th individual derives from using the $i$th mode; $s_{qi}^*$ is the (latent) work-to-home commute stop-making propensity of the $q$th individual should s/he use the mode $I$; $s_{qi}$ is the number of work-to-home commute stops on the choice of mode $i$ to work ($s_{qi}$ is unobserved for the non-chosen modes); $w_{qi}^*$ is the (latent) post home arrival stop-making propensity of the $q$th individual should s/he use the mode $I$; and $w_{qi}$ is the number of work-to-home commute stops on the choice of mode $i$ to work ($w_{qi}$ is unobserved for the non-
chosen work modes). \( s_{qi} \) is characterized by the evening commute stop-making propensity \( s_{qi}^* \) and the threshold bounds (the \( \delta \)'s) in the usual ordered response fashion. A similar relationship exists among \( w_{qi}, w_{qi}^* \), and the threshold bounds represented by \( \theta \)'s. \( z_{qi}, x_{qi}, \) and \( y_{qi} \) are column vectors of exogenous variables, and \( \beta_i, \gamma_i, \) and \( \alpha_i \) are corresponding column vectors of parameters to be estimated. We assume that the \( \epsilon_{qi} \)'s are identically and independently extreme-value distributed (with a location parameter of zero) across alternatives \( i \) and individuals \( q \) (the assumption of independent error distribution across alternatives can be relaxed to accommodate nested logit models of mode choice; the assumption of independence is maintained here for simplicity of presentation). \( \eta_{qi} \) and \( \lambda_{qi} \) are assumed to be identically (and independently) normally-distributed across individuals \( q \) and modes \( i \), each with a standard normal distribution function.

Let \( R_{qi} \) be a dummy variable; \( R_{qi} = 1 \) if the \( ith \) mode is chosen by the \( qth \) individual for his/her work travel, and \( R_{qi} = 0 \) otherwise. Define

\[
\nu_{qi} = \max_{j=1,2,\ldots,k} \nu_{qij} - \nu_{qi}. \tag{Eq. 2}
\]

Equation 1 can now be structured as:

\[
R_{qi}^* = \beta_i^* z_{qi} - \nu_{qi}, \quad R_{qi} = 1 \text{ if } R_{qi}^* > 0, R_{qi} = 0 \text{ otherwise}
\]

\[
s_{qi}^* = \gamma_i x_{qi} + \eta_{qi}, \quad s_{qi} = k \text{ if } \delta_{i,k-1} < s_{qi}^* \leq \delta_{i,k}
\]

\[
w_{qi}^* = \alpha_i y_{qi} + \lambda_{qi}, \quad w_{qi} = l \text{ if } \theta_{i,l-1} < w_{qi}^* \leq \theta_{i,l}
\]

The jointness in the three choices (work mode, number of home-to-work commute stops and number of post home arrival stops) arises because of potential correlation among the random components (\( \nu_{qi}, \eta_{qi}, \lambda_{qi} \)). The key to accommodating these correlations is to transform the random variable \( \nu_{qi} \) into a standard normal random variable \( \nu_{qi}^* \) as follows:

\[
\nu_{qi}^* = \Phi^{-1}\left[F_i(\nu_{qi})\right], \tag{Eq. 4}
\]

where \( \Phi(.) \) is the standard normal distribution function and \( F_i \) is the multinomial distribution function of \( \nu_{qi} \), implied by Equation 2 and the assumed iid extreme value distribution for the
\( \varepsilon_{qi} \)'s. Now, since \( \Phi(v_{qi}^*) = F(v_{qi}) \) by construction (see Equation 4), we can specify a trivariate distribution \( L_3 \) for \( v_{qi}, \eta_{qi}, \) and \( \lambda_{qi} \) having the marginal distributions \( F_j(.) \) for \( v_{qi} \) and \( \Phi(.) \) for \( \eta_{qi} \) and \( \lambda_{qi} \), as:

\[
L_3(v_{qi}, \eta_{qi}, \lambda_{qi}, \rho_{v\eta\lambda}, \rho_{v\lambda\eta}, \rho_{\eta\lambda\eta}) = \Phi_3(v_{qi}^*, \eta_{qi}^*, \lambda_{qi}^*, \rho_{v\eta\lambda}, \rho_{v\lambda\eta}, \rho_{\eta\lambda\eta}),
\]

(Eq. 5)

where \( \Phi_3(.) \) denotes the trivariate normal distribution. From Equations 3 and 5, the joint probability of choosing mode \( i \), number of work-to-home commute stops \( k \), and number of post home arrival stops \( l \) for the individual \( q \) is:

\[
P(R_{qi} = 1, s_{qi} = k, w_{qi} = l) = \\
\Phi_3\left[ \Phi_3^{-1} \left\{ F_j(\beta_i z_{qi}) \right\} \right] \left( \delta_{i,k} - \gamma_i x_{qi} \right), \left( \theta_{i,l} - \alpha_i y_{qi} \right), \rho_{v\eta\lambda}, \rho_{v\lambda\eta}, \rho_{\eta\lambda\eta} \] -
\Phi_3\left[ \Phi_3^{-1} \left\{ F_j(\beta_i z_{qi}) \right\} \right] \left( \delta_{i,k-1} - \gamma_i x_{qi} \right), \left( \theta_{i,l} - \alpha_i y_{qi} \right), \rho_{v\eta\lambda}, \rho_{v\lambda\eta}, \rho_{\eta\lambda\eta} \] -
\Phi_3\left[ \Phi_3^{-1} \left\{ F_j(\beta_i z_{qi}) \right\} \right] \left( \delta_{i,k} - \gamma_i x_{qi} \right), \left( \theta_{i,l-1} - \alpha_i y_{qi} \right), \rho_{v\eta\lambda}, \rho_{v\lambda\eta}, \rho_{\eta\lambda\eta} \] +
\Phi_3\left[ \Phi_3^{-1} \left\{ F_j(\beta_i z_{qi}) \right\} \right] \left( \delta_{i,k-1} - \gamma_i x_{qi} \right), \left( \theta_{i,l-1} - \alpha_i y_{qi} \right), \rho_{v\eta\lambda}, \rho_{v\lambda\eta}, \rho_{\eta\lambda\eta} \],
\]

(Eq. 6)

where

\[
F_j(\beta_i z_{qi}) = \Pr ob (v_{qi} < \beta_i z_{qi}) = \frac{\exp(\beta_i z_{qi})}{\sum_{j=1}^{I} \exp(\beta_j z_{qi})}, i = 1,2,...,I
\]

(Eq. 7)

The parameters to be estimated in the joint model are the \( (K-1) \) \( \delta_{i,k} \) parameters
\( (\delta_{i,0} = -\infty, \delta_{i,K} = +\infty) \), the \( (L-1) \) \( \theta_{i,l} \) parameters \( (\theta_{i,0} = -\infty, \theta_{i,L} = +\infty) \), and the vector
\( (\beta_i, \gamma_i, \alpha_i, \rho_{v\eta\lambda}, \rho_{v\lambda\eta}, \rho_{\eta\lambda\eta}) \) for each mode \( i \) (as structured, \( x_{qi} \) and \( y_{qi} \) do not include a constant).

Defining a set of dummy variables,

\[
M_{qkl} = 1 \text{ if individual } q \text{ makes } k \text{ work-to-home stops and } l \text{ post home arrival stops}
\]

\( (q = 1,2,...Q, k = 1,2,...K, l = 1,2,...L) \)

(Eq. 8)

0 otherwise

the log likelihood function for the estimation of the parameters in the model takes the form
\[
\log L = \sum_{q=1}^{Q} \sum_{i=1}^{I} \left\{ R_{qi} \left( \sum_{k=1}^{K} \sum_{l=1}^{L} M_{qkl} \log[P(R_{qi} = 1, s_{qi} = k, w_{qi} = l)] \right) \right\},
\] (Eq. 9)

If \( \rho_{v_{n,i}}, \rho_{v_{n,i}}, \) and \( \rho_{n,i} \) are equal to zero for each and every mode \( i \), then the likelihood in Equation 9 partitions into a component corresponding to that of a multinomial logit model for mode choice, a second component that represents an independent univariate ordered response model of number of work-to-home commute stops by the chosen work mode, and a third component that represents an independent univariate ordered response model of the number of post home arrival stops by the chosen work mode.

### 5.2 HAZARD-BASED DURATION MODEL

Hazard-based duration models focus on an end-of-duration occurrence (such as home stay or work stay) given that the duration has lasted to some specific time. The concept of conditional probability of “failure” or termination of activity duration recognizes the dynamics of duration; that is, it recognizes that the likelihood of ending a home-stay or work-stay depends on the time since the start of home-stay or work-stay (Bhat 2000). To include an examination of covariates, which affect the duration time, most studies use a proportional hazard model, which operates on the assumption that covariates (exogenous variables) act multiplicatively on some underlying or baseline hazard.

This methodology has been adopted by Bhat (1996) in the modeling of the duration of the shopping activity and by Misra (1999) in the modeling of the home-stay durations before each tour made by non-workers. The econometric modeling structure is presented in the former context here.

Let \( T_q \) represent the continuous activity duration for the individual \( q \) (we consider the time unit of the continuous scale to be in minutes). The hazard for individual \( q \) at some specific time \( u \) on the continuous time scale \( \lambda_q(u) \) is defined using the proportional hazard specification as:

\[
\lambda_q(u) = \lim_{\delta \to 0} \frac{\text{prob}[u + \delta > T_q \geq u | T_q \geq u]}{\delta} = \lambda_0(u) \exp(-\beta'x_{q} + w_{q}),
\] (Eq. 10)
where $\lambda_q(u)$ is the baseline hazard (to be estimated assuming a non-parametric distribution) at time $u$, $x_q$ is a column vector of covariates for individual $q$ (not including a constant), $\beta$ is a column vector of parameters (to be estimated) and $w_q$ is an unobserved heterogeneity term. It can be shown that Equation 10 can be written in the equivalent form,

$$
\ln \Lambda_0(T_q) = \ln \int_0^T \lambda_q(u) du = \beta' x_q + w_q + \varepsilon_q
$$

(Eq. 11)

where $\varepsilon_q$ takes the extreme value form with the distribution function given by:

$$
\Pr(\varepsilon_q < z) = G(z) = 1 - \exp[-\exp(z)]
$$

(Eq. 12)

This continuous duration is not observed; instead, we only observe discrete time intervals (say, 5-minute periods) in which the failure (i.e., end of participation in activity) occurs. Let the discrete time intervals be represented by the index $k$ ($k=1,2,\ldots, K$) and $u^k$ be the continuous time value representing the upper bound of the discrete time period $k$. Therefore,

$$
 k = 1 \text{ if } u \in [0,u^1] \quad k = 2 \text{ if } u \in [u^1,u^2] \quad \ldots, \quad k = K \text{ if } u \in [u^{K-1},\infty].
$$

Let $t_q$ represent the discrete period of failure for individual $q$ (thus, $t_q=k$ if the home- or work-stay duration for individual $q$ ends in period $k$). The objective of the duration model is to estimate the temporal dynamics in duration (that is, how the elapsed time since the start of the activity impacts the future termination of the activity) and the effect of covariates (or exogenous variables) on the continuous activity duration. We can write,

$$
\Pr[t_q = k | w_q] = \Pr[\{u^{k-1} < T_q \leq u^k\} | w_q]
= \Pr[\{\ln \Lambda_0(u^{k-1}) < \ln \Lambda_0(T_q) \leq \ln \Lambda_0(u^k)\} | w_q]
= G(\delta_k - \beta' x_q + w_q) - G(\delta_{k-1} - \beta' x_q + w_q)
$$

(Eq. 13)

where $\delta_k = \ln \Lambda_0(u^k)$

Ignoring the unobserved heterogeneity term $w_q$ in the above equation leads to a simple ordered-response discrete choice structure. This simplifies the estimation and may be an option to consider initially, recognizing that such a procedure may bias parameter estimates and
subsequent duration predictions. The procedure presented below describes the estimation process when the unobserved heterogeneity term $w_q$ is considered and is assumed to be non-parametrically distributed.

We can approximate the distribution of $w_q$ by a discrete distribution with a finite number of support points (say, $S$). Let the location of each support point ($s=1,2…S$) be represented by $l_s$ and let the probability mass at $l_s$ be $\pi_s$. Then, the unconditional probability of an individual $q$’s home- or work-stay “failing” in period $t$ is

$$\text{prob}[l_q = k] = \sum_{s=1}^{S} \left\{ G(\delta_k - \beta' x_q + l_s) - G(\delta_{k-1} - \beta' x_q + l_s) \right\} \pi_s$$  \hspace{1cm} (Eq. 14)

The sample likelihood function for estimation of the location and the probability masses with each of the $S$ support points, and the parameters associated with the baseline hazard (i.e. $(K-1)$ $\delta$ parameters ($\delta_0 = -\infty$ and $\delta_K = +\infty$)) and the covariate effects (the vector $\beta$), can be derived as:

$$L = \prod_{q=1}^{Q} \left\{ \sum_{s=1}^{S} \left\{ \prod_{k=1}^{K} \left\{ G(\delta_k - \beta' x_q + l_s) - G(\delta_{k-1} - \beta' x_q + l_s) \right\}^{M_{qk}} \right\} \pi_s \right\}$$

where

$$M_{qk} = 1 \text{ if failure occurs in period } k \text{ for individual } q \hspace{1cm} (q = 1, 2, ..., Q, k = 1, 2, ..., K)$$

0 otherwise

Since we already have a full set of $(K-1)$ constants represented in the baseline hazard, we impose the normalization that

$$E(w_q) = \sum_{s=1}^{S} \pi_s l_s = 0$$  \hspace{1cm} (Eq. 17)

Our estimation procedure ensures that the cumulative mass over all support points sums to one.
One critical quantity in empirical estimation of the distribution of unobserved heterogeneity is the number of support points, \( S \), required to approximate the underlying distribution. This number is determined by using a stopping rule procedure based on the Bayesian Information Criterion (BIC), which is defined as follows:

\[
BIC = -\ln(L) + 0.5 \cdot R \cdot \ln(N),
\]

(Eq. 18)

where the first term on the right hand side is the log likelihood at convergence, \( R \) is the number of parameters estimated and \( N \) is the number of observations. As the support points are added, the BIC value keeps declining until a point is reached at which the addition of a support point results in an increase in the BIC value. Estimation is terminated at this point, and the number of support points corresponding to the lowest value of BIC is considered the appropriate number for \( S \).

5.3 JOINT DISCRETE/CONTINUOUS ECONOMETRIC SYSTEM

The joint modeling of activity type (discrete choice), activity duration (continuous duration), and travel time (continuous choice) for each stop can be modeled using a discrete/continuous econometric system. Such a methodology has been adopted by Bhat (1998b) for the modeling of post home-arrival activity participation behavior of workers. The estimation procedure presented below uses the full information maximum likelihood estimation technique.

In the following presentation of the model structure, we will use the index \( i \) (\( i=1,2,\ldots,I \)) to represent activity-type choice. The index \( q \) (\( q=1,2,\ldots,Q \)) is used to represent individuals. The equation system can be written as:

\[
\begin{align*}
\hat{u}_{qi} &= \beta_i z_{qi} + \epsilon_{qi}, \text{ activity type } i \text{ is chosen if } \hat{u}_{qi} > \max_{j=1,2,\ldots,I,j \neq i} \hat{u}_{qj} \\
\ln_a q_i &= \theta_i x_{qi} + w_{qi} \text{ for } i = 2,3,\ldots,I \\
\ln_t q_i &= \gamma_i y_{qi} + \eta_{qi} \text{ for } i = 2,3,\ldots,I.
\end{align*}
\]

(Eq.19)

\( \hat{u}_{qi} \) is the indirect (latent) utility that the \( qth \) individual derives from participating in out-of-home activity-type \( I; \) \( \ln_a q_i \) is the logarithm of the activity duration of participation in activity type \( i \) for the \( qth \) individual; and \( \ln_t q_i \) is the logarithm of the travel time duration associated with
participation in activity type $i$ for the $q$th individual. $z_{qi}$, $x_{qi}$, and $y_{qi}$ are column vectors of exogenous variables, and $\beta_i$, $\gamma_i$, and $\alpha_i$ are corresponding column vectors of parameters to be estimated. We assume that the $\varepsilon_{qi}$'s are identically and independently distributed (with a location parameter of zero) across alternatives $i$ and individuals $q$. The $w_{qi}$'s and $\eta_{qi}$'s are assumed to be identically distributed across individuals. We specify a bivariate cumulative normal distribution function $\Phi_{2}(0,0,\sigma_{w_{i}}^{2},\sigma_{\eta_{i}}^{2},\rho_{\eta_{i}w_{i}})$ for $w_{qi}$ and $\eta_{qi}$ in each activity type $i$. $\sigma_{w_{i}}^{2}$ and $\sigma_{\eta_{i}}^{2}$ are the variances of the error terms $w_{i}$ and $\eta_{i}$ respectively and $\rho_{\eta_{i}w_{i}}$ is the correlation between the two error terms.

The continuous variables $a_{qi}$ and $t_{qi}$ are observed if and only if the $ith$ activity type is chosen. Let $R_{qi}$ be a dichotomous variable; $R_{qi} = 1$ if the $ith$ alternative is chosen by the $qth$ individual and $R_{qi} = 0$ otherwise. Defining

$$v_{qi} = \max_{j=1,2,\ldots,J_{i}} u_{qi}^{*} - \varepsilon_{qi},$$  \hspace{1cm} (Eq. 20)

the utility maximizing condition for the choice of the $ith$ alternative may be written as

$$R_{qi} = 1 \text{ if and only if } \beta_{i} z_{qi} > v_{qi} \hspace{1cm} (Eq. 21)$$

Thus, we now have the situation that $a_{qi}$ and $t_{qi}$ are observed if and only if $v_{qi} < \beta_{i} z_{qi}$. The non-normal random variable $v_{qi}$ is transferred into a standard normal random variable as:

$$v_{qi}^{*} = J_{i}(v_{qi}) = \Phi^{-1}\left[F_{i}(v_{qi})\right],$$ \hspace{1cm} (Eq. 22)

where $\Phi(.)$ is the standard normal distribution function. Then Equation 21 can be written as

$$R_{qi} = 1 \text{ if and only if } J_{i}(\beta_{i} z_{qi}) > J_{i}(v_{qi}), or$$

$$R_{qi} = 1 \text{ if and only if } J_{i}(\beta_{i} z_{qi}) > v_{qi}^{*},$$ \hspace{1cm} (Eq. 23)
Let the correlation between $q_i^*$ and $w_{qi}$ be $\rho_{w_i}$ and between $q_i^*$ and $\eta_{qi}$ be $\rho_{\eta_i}$. Combined with the assumed marginal bivariate distribution for $w_i$ and $\eta_i$ and the standard normal distribution of $q_{qi}^*$, we obtain a trivariate normal distribution of $(q_{qi}^*, w_{qi}, \eta_{qi})$ for each activity type $i$ with a mean vector of zero and variance covariance matrix:

$$
\Sigma_i = 
\begin{bmatrix}
1 & \rho_{w_i} \sigma_{w_i} & \rho_{\eta_i} \sigma_{\eta_i} \\
\rho_{w_i} \sigma_{w_i} & \sigma_{w_i}^2 & \rho_{\eta_i} \sigma_{\eta_i} \\
\rho_{\eta_i} \sigma_{\eta_i} & \rho_{\eta_i} \sigma_{\eta_i} & \sigma_{\eta_i}^2 
\end{bmatrix}
$$

(Eq. 24)

The parameters to be estimated are the $\beta_i$ parameters in the activity-type choice model and the following parameters in the activity duration and travel-time duration equations for each activity regime $i$: $(\theta_i, \gamma_i, \sigma_{w_i}, \sigma_{\eta_i}, \rho_{w_i}, \rho_{\eta_i} \text{ and } \rho_{\eta_i \omega_i})$. Define the following quantities for each out-of-home activity type $i$:

$$
g_{qi} = \frac{t_{qi} - \gamma_i y_{qi}}{\sigma_{\eta_i}}, \quad l_{qi} = \frac{d_{qi} - \theta_i x_{qi}}{\sigma_{w_i}}, \quad d_{qi} = \frac{l_{qi} - \rho_{\eta_i \omega_i} g_{qi}}{\sqrt{1 - \rho_{\eta_i \omega_i}^2}} \text{ and } \quad b_{qi} = \Phi^{-1} F_i(\beta_i z_{qi}) - \left[ (\rho_{\eta_i} - \rho_{\eta_i \omega_i} \rho_{w_i}) g_{qi} + (\rho_{w_i} - \rho_{\eta_i \omega_i} \rho_{\eta_i}) l_{qi} \right] \sqrt{1 - \rho_{\eta_i \omega_i}^2} \frac{1 - 2 \rho_{\eta_i} \rho_{w_i} \rho_{\eta_i \omega_i} + \rho_{w_i}^2}{1 - \rho_{\eta_i \omega_i}^2}
$$

(Eq. 25)

The likelihood function to be maximized is:

$$
L = \prod_{q=1}^Q \left[ F_i(\beta_i z_{qi}) \right] \prod_{i=1}^2 \left[ \frac{1}{\sigma_{\eta_i} \sigma_{w_i} \sqrt{1 - \rho_{\eta_i \omega_i}}} \phi(g_{qi}) \phi(d_{qi}) \phi(b_{qi}) \right]^{k_{qi}}
$$

(Eq. 26)

where

$$
F_i(\beta_i z_{qi}) = \frac{\exp(\beta_i z_{qi})}{\sum_{j=1}^1 \exp(\beta_j z_{qi})}
$$

(Eq. 27)
5.4 DISAGGREGATE DESTINATION CHOICE MODEL WITH TIME-BASED PROBABILISTIC CHOICE SET GENERATION

The destination choice model adopted by Misra (1999) uses the travel time distribution by the chosen mode to generate a probabilistic choice set of candidate destinations.

The (logarithm of) travel time to a stop is estimated as a continuous normally distributed variable in Equation 19. The first step in the probabilistic choice set generation for destination choice is to define discrete intervals on this logarithmic scale. The length of the discrete time intervals can vary and can be made as narrow as desired. But the length of each interval should be sufficiently wide to include at least two candidate destination zones from any origin zone. Let there be (K+1) discrete time intervals defined on the logarithmic scale as follows:

\((-\infty, t_1), (t_1, t_2), (t_2, t_3), \ldots, (t_{k-1}, t_k), (t_K, +\infty)\).

Consider an individual \(q\) at a particular zone and let \(t_q\) be the (logarithm of) travel time to his or her next stop. Let \(C_{qk}\) (k=1,2…K) be the set of destinations such that the travel times of the individual from the origin zone to these destinations fall within the interval \((t_{k-1}, t_k)\). By definition each destination \(i\) can belong to one and only one \(C_{qk}\). From the distribution of \(t_q\) determined earlier, we can write the probability of the choice set \(C_{qk}\) as

\[
\pi_{qk} = P(C_{qk}) = \Phi\left(\frac{t_k - E(t_q)}{\sigma_i}\right) - \Phi\left(\frac{t_{k-1} - E(t_q)}{\sigma_i}\right) \tag{Eq. 28}
\]

where \(E(t_q)\) is the expected value of the logarithm of travel-time duration for the \(qth\) individual and \(\sigma_i\) is the estimated standard error of the travel-time duration. By construction,

\[
0 < \pi_{qk} < 1 \forall q, k
\]

and \(\sum_k \pi_{qk} = 1 \forall q \tag{Eq. 29}

Assume that the total utility derived by person \(q\) in choosing destination \(i\) is given by

\[U_{iq} = V_{iq} + \epsilon_{iq}\]

where \(V_{iq}\) represents the systematic utility and \(\epsilon_{iq}\) is the stochastic error term.
assumed to be Gumbel distributed. The conditional probability that destination \( i \) is chosen given the choice set \( C_{qk} \) of which \( i \) is a part therefore takes the multinomial logit form:

\[
P_q(i|C_{qk}) = \frac{\exp(V_{iq})}{\sum_{j \in C_{qk}} \exp(V_{jq})}
\]  

(Eq. 30)

Note: \( P_q(i|C_{qk}) = 0 \) if \( i \not\in C_{qk} \)

The systematic component of the utility function in the above expression is assumed to have a linear-in-parameters form and hence can be written as \( V_{iq} = \beta x_{iq} \). Here, \( x_{iq} \) is the vector of exogenous variables and \( \beta \) is the vector of parameters to be determined.

The unconditional probability of choice of destination is:

\[
P_q(i) = \sum_k P(i|C_{qk})\pi_{qk}.
\]  

(Eq. 31)

Define \( \delta_{iq} = 1 \) if individual \( q \) chose destination \( i \), 0 otherwise. The log likelihood function can therefore be written as

\[
\log L = \sum_q \sum_j \delta_{jq} \log(P_q(j)).
\]  

(Eq. 32)

### 5.5 BIVARIATE BINARY-ORDERED RESPONSE PROBIT MODEL

This methodology is used for the joint modeling of a binary response variable and an ordered response variable. This can be applied in the joint choice modeling of an individual’s decision to participate in at least one out-of-home activity (binary) and the number of stops undertaken over the whole day (ordered variable) (Misra 1999). The joint modeling recognizes the fact that there may be factors common to the propensity to go out and the propensity to make many stops. The modeling methodology presented here is in this context.

Let the index \( i \) represent the \( ith \) person (\( i=1,2,\ldots I \)). Define a binary variable \( o_i \) and an integer variable \( s_i \) as
\[ a_i = \begin{cases} 0 & \text{if person } i \text{ makes no trips} \\ 1 & \text{if person } i \text{ makes at least one trip} \end{cases} \]
\[ s_i = k \quad \text{if person } i \text{ makes } k \text{ stops and } k \leq K-1 \]
\[ K \quad \text{if person } i \text{ makes } K \text{ or more stops in the day} \quad \text{(Eq. 33)} \]

where, \( K \) is a pre-defined integer that represents the maximum number of stops that may occur in an individual’s activity string.

The equation system can be expressed as

\[
\begin{align*}
o_i^* &= \beta^* x_i + \epsilon_i, \quad o_i = 1 \text{ if } o_i^* > 0, \quad o_i = 0 \text{ if } o_i^* \leq 0 \\
s_i^* &= \gamma^* z_i + \nu_i, \quad s_i = k \text{ if } \psi_{k-1} < s_i^* \leq \psi_k, \quad k = 1,2,3,...,K \quad \text{(Eq. 34)}
\end{align*}
\]

\( o_i^* \) is the latent propensity for non-worker \( i \) to make at least one out-of-home activity stop and \( s_i^* \) is the (latent) stop making propensity of the \( i \)th individual should s/he decide to make at least one out-of-home activity stop. \( s_i \) is characterized by the stop-making propensity \( s_i^* \) and the threshold bounds (the \( \psi \)’s) in the usual ordered response fashion. \( z_i \) and \( x_i \) are column vectors of exogenous variables, and \( \beta \) and \( \gamma \) are corresponding column vectors of parameters to be estimated. We assume that the stochastic error terms \( \epsilon_i \) and \( \nu_i \) are each standard normal distributed for each person (\( \Phi(.) \) is the standard normal distribution function). Let \( \rho \) be the covariance between the two error terms for each person. Hence, the joint distribution of the two random variables \( \epsilon_i \) and \( \nu_i \) has a bivariate normal distribution (\( \Phi_2(.) \) is the bivariate normal distribution function), with means of the two variables equal to zero, variance equal to 1 and the covariance between the two variables equal to \( \rho \).

The unconditional probability of stop making can therefore be expressed as

\[
\begin{align*}
\text{Prob}(s_i = 0) &= 1 - \Phi(\beta^* x_i) \\
\text{Prob}(s_i = k) &= \Phi_2(\beta^* x_i, -\gamma^* z_i + \psi_k, \rho) - \Phi_2(\beta^* x_i, -\gamma^* z_i + \psi_{k-1}, \rho) \quad \text{(Eq. 35)}
\end{align*}
\]
for \( k = 1,2,3,...,K \)

Based on construction, the following relations hold:
\[ \Phi_z(\beta x_i - \gamma' z_i + \psi_0, \rho) = 0 \]  
\[ \Phi_z(\beta x_i - \gamma' z_i + \psi_K, \rho) = \Phi(\beta' x) \]  
(Eq. 36)

The parameters to be estimated in the joint model are the \((K-1)\) \(\psi_k\) parameters \((\psi_0 = -\infty, \psi_K = +\infty)\) and the vector \((\beta, \gamma, \rho)\). Defining a set of dummy variables,

\[ \delta_{k,i} = 1 \text{ if individual } i \text{ makes } k \text{ stops} \]  
\[ (i = 1,2,...I, k = 1,2,...K) \]  
(Eq. 37)

0 otherwise

the log likelihood function for the estimation of the parameters in the model takes the form

\[ \log L = \sum_{i=1}^{I} \sum_{k=1}^{K} \delta_{k,i} \ln[p(s_i = k)] \]  
(Eq. 38)

If the covariance term is zero, the likelihood function decomposes into two components: one for modeling the choice of making at least one out-of-home activity stop (a binary probit model) and a second for modeling of the number out-of-home activity stops (an ordered response probit model).

5.6 SIMULTANEOUS EQUATION SYSTEM

The simultaneous equation system can be used for the joint modeling of two or more continuous variables. Misra applied this methodology for the joint modeling of travel time and activity duration for non-workers (1999). The model structure is discussed in this context here.

In the following presentation of the model structure, we will use the index \(k\) \((k=1,2,...K)\) to represent activity-type. The index \(i\) \((i=1,2,...I)\) is used to represent individuals. The travel time and the activity duration for any individual \(i\) can be written as:

\[ t_{ik} = \gamma' x_{ik} + \eta_{ik} \]
\[ a_{ik} = \theta' y_{ik} + w_{ik} \]  
(Eq. 39)

\( for \ k = 1,2,3,...,K \)
$t_{ik}$ is the logarithm of the travel time duration associated with participation in activity type $k$ for the $ith$ individual and $a_{ik}$ is the logarithm of the activity duration of participation in activity type $k$ for the $ith$ individual. $x_{ik}$ and $y_{ik}$ are column vectors of exogenous variables and $\gamma_k$ and $\theta_k$ are corresponding column vectors of parameters to be estimated. The stochastic error terms $w_{ik}$ and $\eta_{ik}$ are assumed to be identically distributed across individuals. We also assume that $w_k$ and $\eta_k$, corresponding to activity $k$, have a bivariate cumulative normal distribution with parameters given by $\Phi_2(0,0,\sigma_{wk}^2,\sigma_{\eta k}^2,\rho_{wk,\eta k})$, where $\sigma_{wk}^2$ and $\sigma_{\eta k}^2$ are the variances of the error terms respectively and $\rho_{wk,\eta k}$ is the correlation between the two error terms.

The vector of parameters to be estimated is $(\theta_k,\gamma_k,\sigma_{wk},\sigma_{\eta k} and \rho_{wk,\eta k})$. Define the following quantities for each out of home activity type $k$:

$$g_{ik} = \frac{t_{ik} - \gamma_k y_{ik}}{\sigma_{\eta k}}, \quad l_{ik} = \frac{a_{ik} - \theta_k x_{ik}}{\sigma_{wk}} \tag{Eq. 40}$$

The probability that individual $i$, who performs activity type $k$, has travel time $t_{ik}$ and activity duration $a_{ik}$ is given by

$$P_{ik}(g_{ik},l_{ik}) = \Phi_2(g_{ik},l_{ik},\rho_{wk,\eta k}) \tag{Eq. 41}$$

Defining a dummy variable $\delta_{ik}$ that takes a value 1 if individual $i$ participates in activity type $k$ and 0 otherwise, the log likelihood function to be maximized is:

$$Log L = \sum_{i=1}^{I} \sum_{k=1}^{K} \delta_{ik} \log[P_{ik}(g_{ik},l_{ik})] \tag{Eq. 42}$$
5.7 MODELING STOP FREQUENCY BY ACTIVITY TYPE FOR NON-WORKERS

The methodology presented here is adopted in the modeling of the number of stops by activity type for non-workers, given the total number of stops made by the individual over the day (Misra 1999).

Let the probability that an individual \( i \) makes an activity stop of type \( t \) be \( R_{it} \). If the total number of activity types is \( T \), this probability can be determined from a multinomial logit model as follows:

\[
R_{it} = \frac{\exp(\beta_{it} w_i)}{\sum_{j=1}^{T} \exp(\beta_{ij} w_j)}, \quad \text{(Eq. 43)}
\]

where \( w_i \) is a column vector of exogenous variables and \( \beta_{it} \) is the vector of parameters to be determined for each activity type, except for a base category.

To relate the probability of stop making by activity type with the individual’s stop allocation (which is what is observed), we assume that households follow a zero-order process in assigning stops to activity types (i.e., the successive assignments of stops to activity-type categories constitutes an independent sequence, after controlling for exogenous variables). Let \( S_{it} \) be the total number of stops of activity type \( t \) made by an individual \( i \) and \( S_i (> 0) \) be the total number of stops of all activity types made by the individual \( i \). Then \( \sum_i S_{it} = S_i \).

The probability of a non-worker making \( k_1 \) stops of activity type 1, \( k_2 \) stops of activity type 2… \( k_T \) stops of activity type \( T \) (given that the total number of stops made by the individual is \( k \) ) can be given by

\[
P(S_{i1} = k_1, S_{i2} = k_2, \ldots, S_{iT} = k_T \mid S_i = k, k > 0) = \frac{k!}{\prod k_i} \prod R_{it}^{k_i} \quad \text{(Eq. 44)}
\]
Define the following dummy variables:

\[ \eta_{ik} = \begin{cases} 1 & \text{if } S_i = k \\ 0 & \text{otherwise} \end{cases} \]

\[ \delta_{k_1, k_2, \ldots, k_r} = \begin{cases} 1 & \text{if } S_{i1} = k_1, S_{i2} = k_2, \ldots, S_{ir} = k_r \\ 0 & \text{otherwise} \end{cases} \]  
(Eq. 45)

The log likelihood function can therefore be written as

\[
\log L = \sum_{i=1}^{l} \sum_{k=1}^{K} \eta_{ik} \sum_{k_1=1}^{k} \sum_{k_2=1}^{k} \ldots \sum_{k_r=1}^{k} \delta_{k_1, k_2, \ldots, k_r} \left( \log k! - \log \prod_i k_i! \right) + \sum_i k_i R_{ii} \]

(Eq. 46)

5.8 MODELING STOP SEQUENCING FOR NON-WORKERS

Modeling the stop-sequence in the activity string of a non-worker requires the determination of the number of in-home episodes (and hence the number of tours, since the total number of activity stops is known), the number of stops in each tour and the activity type for each activity stop. The methodology presented here is based on Misra (1999).

Consider a non-worker who makes a total of \( s \) stops of which \( k_1 \) stops are of activity type 1, \( k_2 \) stops are of activity type 2… \( k_T \) stops are of activity type \( T \). Let the set of all feasible pattern strings (all permutations of \( k_1 \) stops of activity type 1, \( k_2 \) stops of activity type 2… \( k_T \) stops of activity type \( T \) and each possible value \( r \) of intermediate home episodes \([r=0,1,2…s-1]\) that do not contain two consecutive in home episodes) for the individual be represented by \( \vartheta \). The first and last activity episodes of all possible pattern strings correspond to in-home activities and hence are not classified as intermediate in-home activity stops. Let a single member of this feasible choice set be \( g \).

Define a binary variable \( A'_g = 1 \) if pattern \( g \) has \( r \) intermediate in-home episodes, \( A'_g = 0 \) otherwise. The utility attributable to pattern string \( g \) due to the number of in-home activity stops can be given by
\[ v_g = \sum_{r=0}^{s-1} A_g^r v_r, \quad \text{(Eq. 47)} \]

where, \( v_r = \beta_r x \)

\( v_r \) is the utility assigned to making \( r \) intermediate in-home activity episodes, \( x \) is the vector of exogenous variables affecting stop making, and \( \beta_r \) is the vector of parameters to be determined.

Let the number of stops in tour \( h \) (\( h=1,2...r+1 \)) of the individual’s activity pattern \( g \) be \( s_h \).
Define a binary variable \( B_h^q = 1 \) if there are \( q \) stops in tour \( h \) of pattern \( g \), \( B_h^q = 0 \) otherwise. The utility associated with the stop distribution among the tours for pattern \( g \) is

\[ w_g = \sum_{h \in M_g} \sum_q B_h^q \gamma_h^q, \quad \text{(Eq. 48)} \]

where \( \gamma_h^q \) is the utility associated with assigning \( q \) stops to tour \( h \) in pattern string \( g \) and \( M_g \) is the set of all tours in pattern string \( g \).

Let the total number of activity episodes performed by the person be \( E \) (\( E = r+s+2 \)). Let the activity pattern string be represented by \((a_1,a_2,a_3,...a_r,a_{r+1},...,a_{E})\), where \( a_e \) represents the activity type of episode \( e \). Let \( \delta_n(a_e) = 1 \) if the activity type of episode \( e \) is \( n \) (\( n=1,2...T+1 \)), 0 otherwise. Let \( \lambda_{mn} \) be the constant utility derived by performing activity \( n \) immediately after activity \( m \) and \( \Delta_a \) be the additional utility derived by performing activity \( a \) as the first stop of the day. The Markov utility derived from the pattern string \( g \) can therefore be written as

\[ \pi_g = \sum_{e=2}^E \left[ \sum_{m=1}^{T+1} \sum_{n=1}^{T+1} \delta_m(a_{e-1}) \delta_n(a_e) \lambda_{mn} \right] + \sum_{n=1}^{T+1} \delta_n(a_2) \Delta_n \quad \text{(Eq. 49)} \]

The total utility \( \tilde{\theta}_g \) derived from this pattern string can therefore be expressed as

\[ \tilde{\theta}_g = v_g + w_g + \pi_g \]
The probability that a non-worker will choose a particular pattern string \( g \) from among all possible strings \( \vartheta \) can be given by the multinomial logit formulation as

\[
\text{Prob}_i(g) = \frac{\exp(\tilde{\theta}_g)}{\sum_{g \in \vartheta} \exp(\tilde{\theta}_g)} \quad \text{(Eq. 50)}
\]

Define \( \tau_{gi} = 1 \) if person \( i \) chose pattern string \( g \), 0 otherwise. The log likelihood function can therefore be written as

\[
\log L = \sum_i \sum_{g \in \vartheta} \tau_{gi} \log(\tilde{P}_{gi}) \quad \text{(Eq. 51)}
\]
CHAPTER 6. SUMMARY

There has been an increasing realization in the travel-demand modeling field that the conventional trip-based approach needs to be replaced with an activity-based approach that is behaviorally oriented. Several comprehensive activity-based systems have been developed. The current research aims at advancing the state of the art in activity-based modeling by addressing the activity patterns of both workers and non-workers within a household context.

This report presents a comprehensive representation framework for travel demand modeling using the activity-based approach. The framework identifies the key agents and their inter-relationships. Household decisions about employment, household location, and auto ownership are classified as medium-term decisions. The daily activity-travel patterns are classified as short-term decisions. The framework adopts a two-level structure in which the medium-term decisions are modeled prior to the modeling of short-term decisions.

This report also characterizes the activity patterns of workers and non-workers. A conceptual framework developed captures the inter-personal dependencies and resource sharing within the households that constrain the activity-travel patterns of individuals. These constraints are largely dependant on the household structure and it is very difficult to develop a single model to capture the interactions in all types of households. Hence, five major types of households were identified based on an empirical analysis of data from the Dallas Fort Worth area. Representation frameworks were then developed separately for these household types.

This report also describes the approach that has been adopted for the modeling of medium-term household decisions and the mathematical structures of the modeling methods to be used were also presented.
REFERENCES


